

# Epi Methods Subcommittee Webinar: Skill Refresher: Calculation of Tests of Trends in Proportions and Rates for Public Health

Thursday, March 29 at 2pm ET



Council of State and Territorial Epidemiologists

# Housekeeping



- Mute your lines
- The slides and session recording will be archived to the CSTE Webinar Library
- Use the chat box feature to ask the presenter questions
- Complete your evaluation after the webinar
  - Webinar feedback
  - Ideas for future webinars

# Announcements



- Registration **for the 2018 Annual Conference** opens February 1<sup>st</sup>
- Next **Spatial Analysis Workgroup Call** will be held on Tuesday, April 17<sup>th</sup> at 1pm ET, contact [Jarrazola@cste.org](mailto:Jarrazola@cste.org) for more details
- Next **Public Health and Health Care Analytics Workgroup Call** will be held on Thursday, April 12<sup>th</sup> at 1pm ET, contact [Jarrazola@cste.org](mailto:Jarrazola@cste.org)

# Objectives



1. State why trends in proportions are important
2. Calculate trend-related measures using free web applications
3. Calculate trend-related measures using R, that can be expanded for other “real world” production projects.

# Presenter



Michael Samuel, DrPH, Data Scientist/Senior Epidemiologist at the California Department of Public Health

# Refresher on Assessing Trends in Proportions (and Related Issues)

 CSTE™ Training session  
Thursday, March 29, 2018

Michael C. Samuel, Dr.P.H.  
Data Scientist / Senior Epidemiologist  
Fusion Center for Strategic Development and External Relations  
California Department of Public Health

# Outline

- All materials at: <http://www.goo.gl/k9YmXJ>
- Last time
  - Some theory and formulas
  - Calculation of simple confidence intervals for proportions, by “hand”, in Excel, in Open Epi, and in R
  - Comparing proportions and Rates
- Chi-Squared tests theory
- Chi-Squared trend test
  - What, why and how
- A note on confidence intervals
- Interactive application in R
- Resources

community	cause	pop	deaths	rate
Jefferson	All Causes	6278	169	2691.9
Jefferson	Alzheimer's	6278	9	143.4
Jefferson	Cardiovascular	6278	59	939.8
River	All Causes	4660	43	922.7
River	Alzheimer's	4660	3	64.4
River	Cardiovascular	4660	15	321.9
Rose	All Causes	2312	38	1643.6
Rose	Alzheimer's	2312	2	86.5
Rose	Cardiovascular	2312	15	648.8

community	surveyPop	highSchool	percent at least high school
Jefferson	622	578	92.9%
River	460	216	47.0%
Rose	229	224	97.8%

Number of Sugary Groups Eaten Last	Obese		Total	% Obese
	Yes	No		
1	1	8	9	11.1%
2	2	8	10	20.0%
3	4	7	11	36.4%
4	8	8	16	50.0%
Total	15	31	46	32.6%

Favorite Vegy	Obese		Total	% Obese
	Yes	No		
Broccoli	1	8	9	11.1%
Carrots	2	9	11	18.2%
Lettuce	8	7	15	53.3%
Tomato	4	7	11	36.4%
Total	15	31	46	32.6%

# Chi-Squared tests

- Used to assess association (versus independence) in a contingency (“cross tabulation”) table
- Assess whether or not observed counts agree with expected counts, based on assumption of independence
- Can be used for
  - “2 by 2” table (two proportions, Relative Risk, Odds Ratio)
  - N x K table
  - 2 by K table with ordered values (trend test)

# Chi-Squared Tests

- In general, the higher the chi-square value, the greater the likelihood there is a statistically significant difference between the groups you are comparing
- To know for sure, you need to determine the p-value with your software or in a chi-square table
- Many variants and nuances to the simple tests

# General Form/Idea of Chi-Squared Statistic

$O_{11}$	$O_{12}$	.	.	<b>N1. = Row 1 Total</b>
$O_{21}$	$O_{ij}$	.	.	<b>N2. = Row 2 Total</b>
.	.	$O_{ij}$ 	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	$O_{64}$	.
<b>N.1 = Col</b>	.	.	.	<b>N.. = Grand Total</b>

- $O_{ij}$  = observed count
- $E_{ij}$  = Expected count =  $\frac{\text{Row } i \text{ total} * \text{Column } j \text{ total}}{\text{Grand Total}} = \frac{N_{i.} * N_{.j}}{N..}$
- $\chi^2 = \sum_{ij} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
- Compare to  $\chi^2$  with (number of rows-1)\*(number of columns -1) “degrees of freedom”

Observed		High School Diploma?			
		Yes	No		
Community	Jefferson	578	44	622	92.9%
	River	224	5	229	97.8%

		Yes	No		Expected	
Jefferson River				622	586.2	35.8
				229	215.8	13.2
		802	49	851		

$\xrightarrow{\frac{N_{i.} * N_{.j}}{N_{..}}}$

Chi-squared contribution:

$\frac{(O - E)^2}{E}$	$\frac{(O - E)^2}{E}$
$\frac{(O - E)^2}{E}$	$\frac{(O - E)^2}{E}$

0.11	1.87
0.31	5.08

$$\Sigma = 7.38$$

Start Enter Results Examples Help

Clear Settings Conf. level=95% Calculate

Add Stratum Stratum 1 Delete Stratum

**Open Epi 2 x 2 Table**

		Disease		Totals
		(+)	(-)	
Exposure	(+)	578	44622	
	(-)	224	5229	
Totals		802	49851	

## 2 x 2 Table Statistics

### Single Table Analysis

		Disease	
		(+)	(-)
Exposure	(+)	578	44622
	(-)	224	5229
		802	49851

### Chi Square and Exact Measures of Association

Test	Value	p-value(1-tail)	p-value(2-tail)
Uncorrected chi square	7.377	0.003303	0.006605
Yates corrected chi square	6.504	0.005383	0.01077
Mantel-Haenszel chi square	7.369	0.003318	0.006637
Fisher exact		0.003088(P)	0.006176
Mid-P exact		0.001953(P)	0.003906

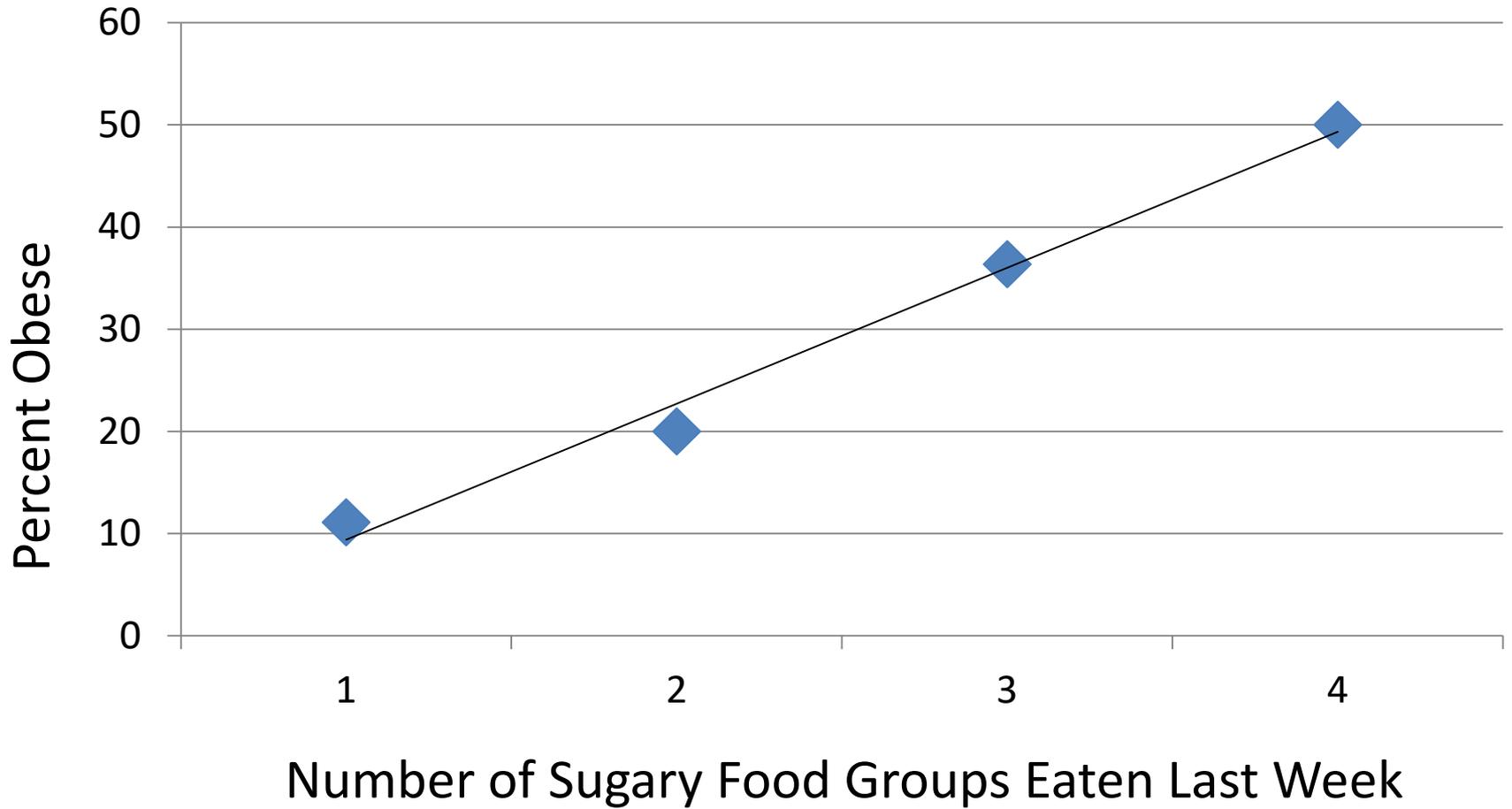
# Types of Statistical Tests

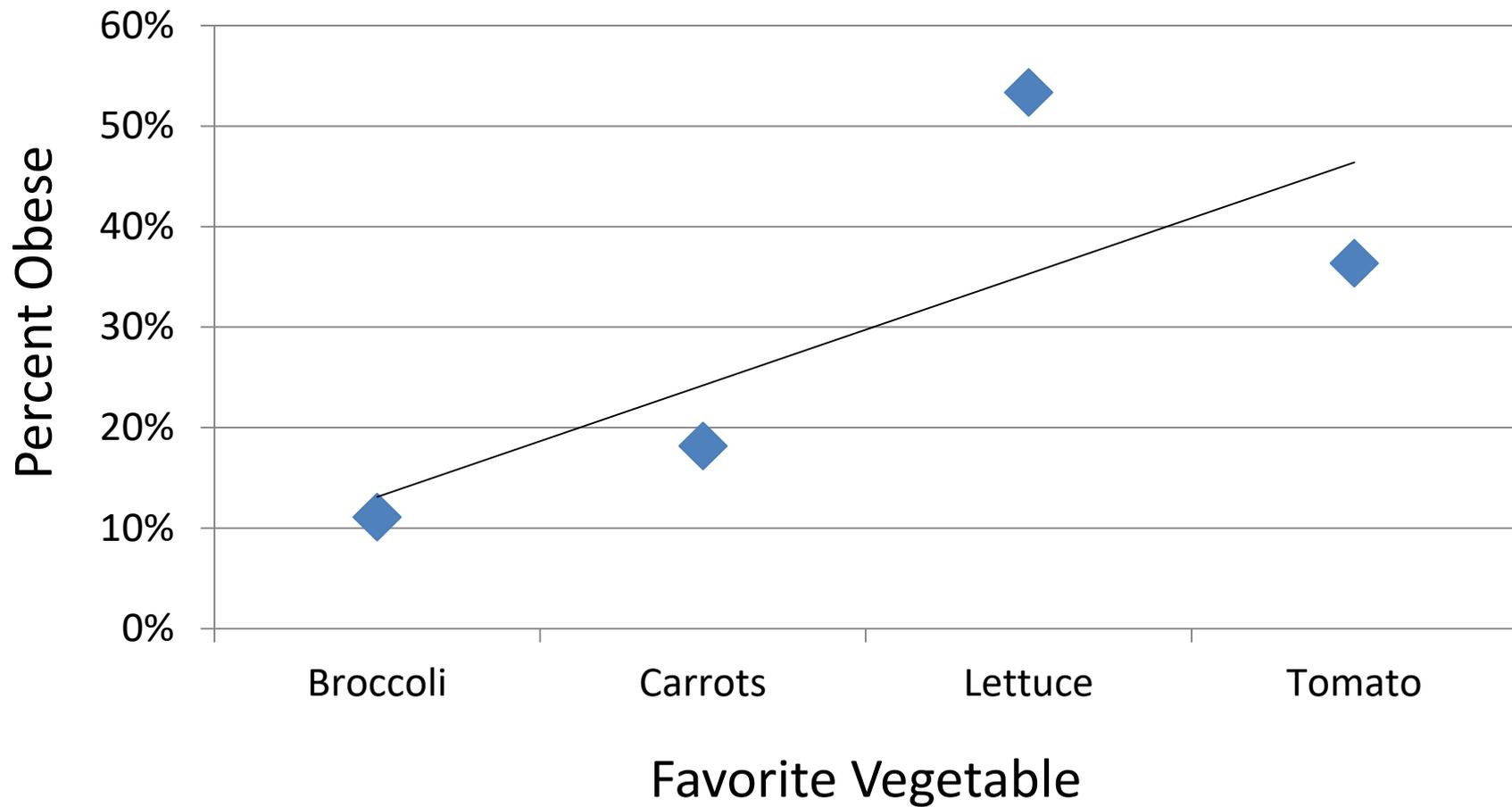
<b>Parade of Statistics Guys</b>	
<i>The right test...</i>	<i>To use when...</i>
Pearson chi-square (uncorrected)	Sample size > 100 Expected cell counts > 10
Yates chi-square (corrected)	Sample size > 30 Expected cell counts $\geq 5$
Mantel-Haenszel chi-square	Sample size > 30 Variables are ordinal
Fisher's exact test	Sample size < 30 and/or Expected cell counts < 5

<https://sph.unc.edu/nciph/focus/>

Number of Sugary Groups Eaten Last	Obese		Total	% Obese
	Yes	No		
1	1	8	9	11.1%
2	2	8	10	20.0%
3	4	7	11	36.4%
4	8	8	16	50.0%
Total	15	31	46	32.6%

Favorite Vegy	Obese		Total	% Obese
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# Chi-squared Trend Test

- Test of linear trend in series of proportions
  - or Relative Risks or Odds Ratios
- “Cochran–Armitage test for trend”
- “(Extended) Mantel-Haenszel chi square for linear trend”
- Formula is more complex than general chi-squared
  - <http://www.bmj.com/about-bmj/resources-readers/publications/statistics-square-one/8-chi-squared-tests>
  - [https://en.wikipedia.org/wiki/Cochran%E2%80%93Armitage\\_test\\_for\\_trend](https://en.wikipedia.org/wiki/Cochran%E2%80%93Armitage_test_for_trend)
- Algebraically identical to simple test of slope

http://www.openepi.com

**Open Source Epidemiologic Statistics for Public Health**

Now in Version 3

- Proportion
- Two by Two Table
- Dose-Response**
- R by C Table
- Matched Case Control

...stratified analysis with exact tests, random numbers, sensitivity, and links to other useful sites.

...web server or downloaded and encrypted and HTML, and should be noted that you are seeing this, your browser and Android cellphones

...always a good idea to check the calculations are provided.

...d. Some of the components are available in full text at [Bill and Melinda Gates](#)

[Mason2006.cdc.gov/abstracts](#). OpenEpi development was supported in part by a grant from the [Foundation to Emory University, Rollins School of Public Health](#).

A toolkit for creating new modules and for translation is included. Please let us know if you would like to collaborate in this way. Suggestions, comments, and expressions of interest in contributing to this effort should be sent by email to: [andy.dean@gmail.com](mailto:andy.dean@gmail.com), [cdckms@sph.emory.edu](mailto:cdckms@sph.emory.edu), and [msoc@cdc.gov](mailto:msoc@cdc.gov)

Suggested citation: Dean AG, Sullivan KM, Soe MM. OpenEpi: Open Source Epidemiologic Statistics for Public Health, Version 3. [www.OpenEpi.com](http://www.OpenEpi.com), updated 2013/04/06, accessed 2018/03/25.

Start Enter Results Examples Help

Clear Calculate

Add Stratum Stratum 1 ▼ Delete Stratum

**Dose Response in a Case Control Study**

Exposure Level	Cases	Controls	Total
0(Baseline)	1	8	9
1	2	8	10
2	4	7	11
3	8	8	16
Total	15	31	46

## Dose Response Analysis

Stratum 1					
Exposure Level	Cases	Controls	Total	Odds of Exp.	Odds Ratio
0	1	8	9	0.13	1
1	2	8	10	0.25	2
2	4	7	11	0.57	4.57
3	8	8	16	1	8
Total	15	31	46		

Mantel-Haenszel Summary Odds Ratios and Crude OR for Each Exposure Level		
Exposure	MH Summary OR	Crude OR
Level 0 vs. Level 0:	1	1
Level 1 vs. Level 0:	2	2
Level 2 vs. Level 0:	4.571	4.571
Level 3 vs. Level 0:	8	8

If MH and crude ORs are equal, confounding by the stratifying variable was not present and stratification is unnecessary.

Extended Mantel-Haenszel chi square for linear trend=  
p-value(1 degree of freedom)=

4.16  
0.04150

# Epi Info Stat Calc

<https://www.cdc.gov/epiinfo/index.html>

The image shows two overlapping windows from the Epi Info 7 software. The background window is the 'Epi Info 7 - Menu' window, which has a menu bar with 'File', 'View', 'Tools', 'StatCalc', and 'Help'. The 'StatCalc' menu is open, showing options: 'Sample Size and Power', 'Chi Square for Trend' (circled in red), 'Tables (2 x 2 x n)', 'Poisson (rare event vs. std.)', 'Binomial (proportion vs. std.)', 'Matched pair case control s', and 'OpenEpi.com'. The foreground window is titled 'StatCalc - Chi Square for Trend' and displays the results of an 'Analysis For Linear Trends In Proportions'. It contains a table with columns for 'Exposure Score', 'Cases', 'Controls', and 'Odds Ratio'. Below this table is an 'Add Row' button and a summary table with two rows: 'Chi Square for linear trend (Extended Mantel-Haenszel)' with a value of 4.15547, and 'p value' with a value of 0.04150.

Exposure Score	Cases	Controls	Odds Ratio
1	1	8	1.000
2	2	8	2.000
3	4	7	4.571
4	8	8	8.000

Chi Square for linear trend (Extended Mantel-Haenszel)	4.15547
p value	0.04150

# Trend Test - SAS

```
proc freq data=data.trendexample order=formatted;
```

```
tables grands*ident/ nopercnt nocol chisq trend;
```

```
run;
```

Table of grands by ident			
grands(grands)	ident(ident)		
Frequency Row Pct	1 Yes	2 No	Total
1	1 11.11	8 88.89	9
2	2 20.00	8 80.00	10
3	4 36.36	7 63.64	11
4	8 50.00	8 50.00	16
<b>Total</b>	15	31	46

Statistics for Table of grands by ident

Statistic	DF	Value	Prob
Chi-Square	3	4.8889	0.1801
Likelihood Ratio Chi-Square	3	5.1980	0.1579
<b>Mantel-Haenszel Chi-Square</b>	<b>1</b>	<b>4.7349</b>	<b>0.0296</b>
Phi Coefficient		0.3260	
Contingency Coefficient		0.3100	
Cramer's V		0.3260	
<b>WARNING: 38% of the cells have expected counts less than 5. Chi-Square may not be a valid test.</b>			

Sample Size = 46

Cochran-Armitage Trend Test	
Statistic (Z)	<b>2.2000</b>
One-sided Pr > Z	0.0139
Two-sided Pr >  Z	<b>0.0278</b>

Sample Size = 46

# Trend Test - SPSS

## CROSSTABS

/TABLES=grands BY ident  
 /FORMAT=AVALUE TABLES  
 /STATISTICS=CHISQ  
 /CELLS=COUNT  
 /COUNT ROUND CELL.

grands \* ident Crosstabulation

Count		ident		Total
		0	1	
grands	1	8	1	9
	2	8	2	10
	3	7	4	11
	4	8	8	16
Total		31	15	46

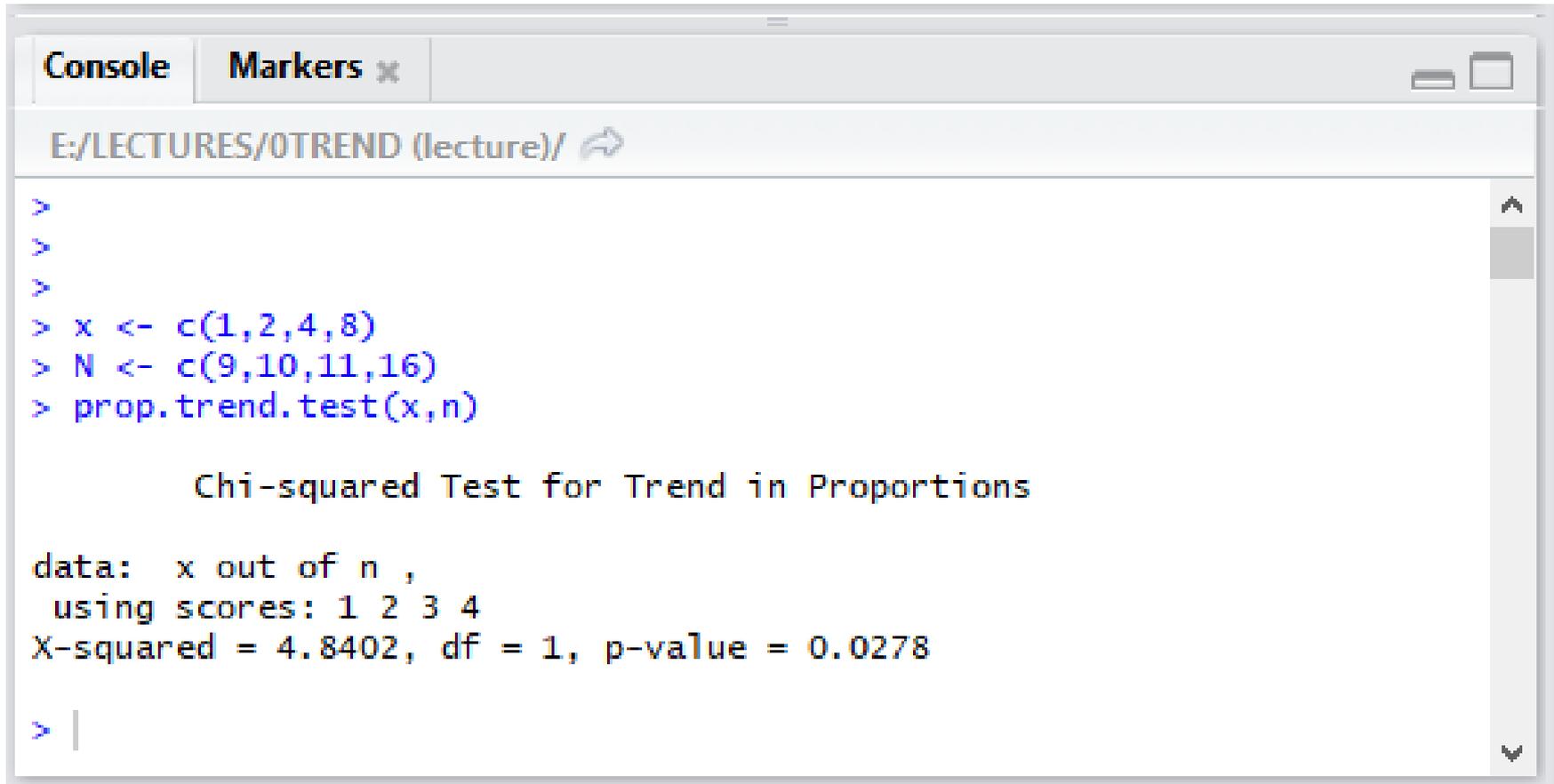
## Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	4.889 <sup>a</sup>	3	.180
Likelihood Ratio	5.198	3	.158
Linear-by-Linear Association	4.735	1	.030
N of Valid Cases	46		

a. 3 cells (37.5%) have expected count less than 5. The minimum expected count is 2.93.

R: (at <http://www.r-project.org/>)

R Studio Interface: <http://www.rstudio.com/>



```
Console Markers x E:/LECTURES/0TREND (lecture) ↗  
>  
>  
>  
> x <- c(1,2,4,8)  
> N <- c(9,10,11,16)  
> prop.trend.test(x,n)  
  
Chi-squared Test for Trend in Proportions  
  
data: x out of n ,  
using scores: 1 2 3 4  
X-squared = 4.8402, df = 1, p-value = 0.0278  
  
> |
```

## Sweets and Obesity Example - Differences in $X^2$ and p-value

Software	$X^2$ trend	P-value	reference
R	4.84	0.028	?
SAS ("chi")	4.73	0.0296	"Mantel-Haenszel"
SAS ("trend")	4.84 (2.20 <sup>2</sup> )	0.0278	"Cochran-Armitage"
SPSS	4.735	0.030	?
Open Epi	4.16	0.042	"Extended Mantel Haenszel Chi Square for linear trend" (Schlesselman, JJ. Case-Control Studies: Design, Conduct, Analysis. Oxford Univ. Press, NY, 1982; p.200-206)
Stat Calc	4.16	0.042	"Mantel extension of the Mantel-Haenszel summary odds ratio and chi square"

## Original Contributions

# Sexual Practices and Risk of Infection by the Human Immunodeficiency Virus

## The San Francisco Men's Health Study

Warren Winkelstein, Jr, MD, MPH; David M. Lyman, MD, MPH; Nancy Padian, MS, MPH;  
Robert Grant, MPH; Michael Samuel; James A. Wiley, PhD; Robert E. Anderson, MD;  
William Lang, MD

The San Francisco demiology and a cohort of 103 probability seropositivity rate homosexual/bis pants were HIV male sexual pa was 17.6%. For Only receptive infection. Douc significantly to r

AMONG homo large numbers receptive anal been the most risk factors for viruses associat immunodeficien However, all of studies of risk infection have clinical or volun

From the School of and Lyman, Ms Padian and the Survey Resear of California, Berkeley Francisco (Drs Anders ettsial Disease Labor Health Services, Berkeley Research Institute, Urisco (Dr Levy).  
Reprint requests to city of California, Berk

JAMA, Jan 16, 1987

Table 1.—Association of Number of Male Sexual Partners in Previous Two Years and Human Immunodeficiency Virus (HIV) Serologic Status\*

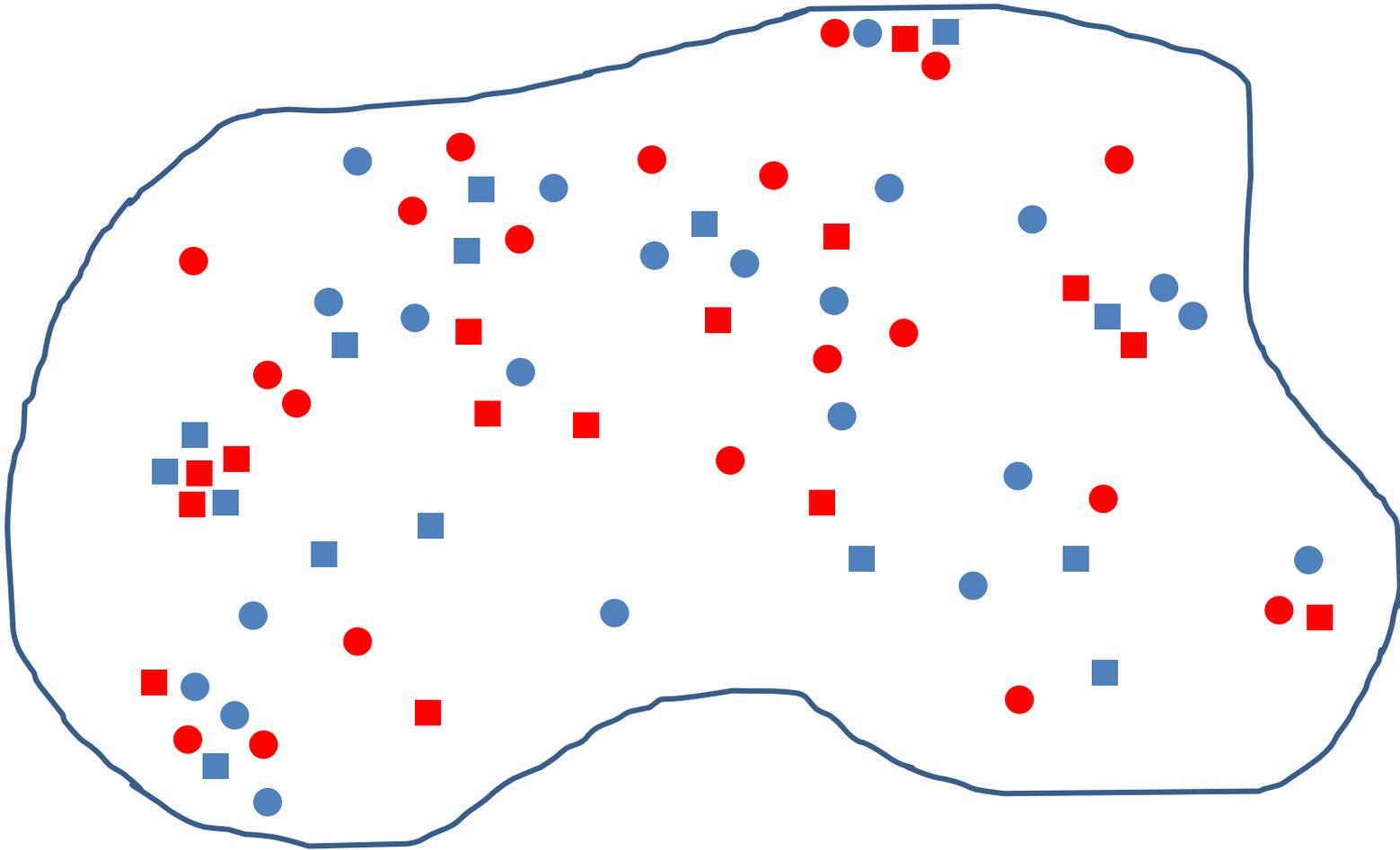
No. of Male Partners	Study Sample		Population	
	No. Examined	% HIV Antibody Positive	% HIV Antibody Positive†	95% Confidence Interval
None	17	17.6	19.2	5.2-41.5
1	66	18.2	17.9	9.5-29.0
2-9	206	31.6	31.9	25.2-39.0
10-49	312	53.8	53.7	47.4-59.6
≥50	195	70.8	70.5	62.7-76.8
<b>Total</b>	<b>796</b>	<b>48.5</b>	<b>48.2</b>	<b>44.3-52.0</b>

\*Subjects with missing data (n = 13) were excluded. The  $\chi^2$  for trend of the association of number of partners and HIV antibody seropositivity in the sample is 86.7,  $P < .0001$ .

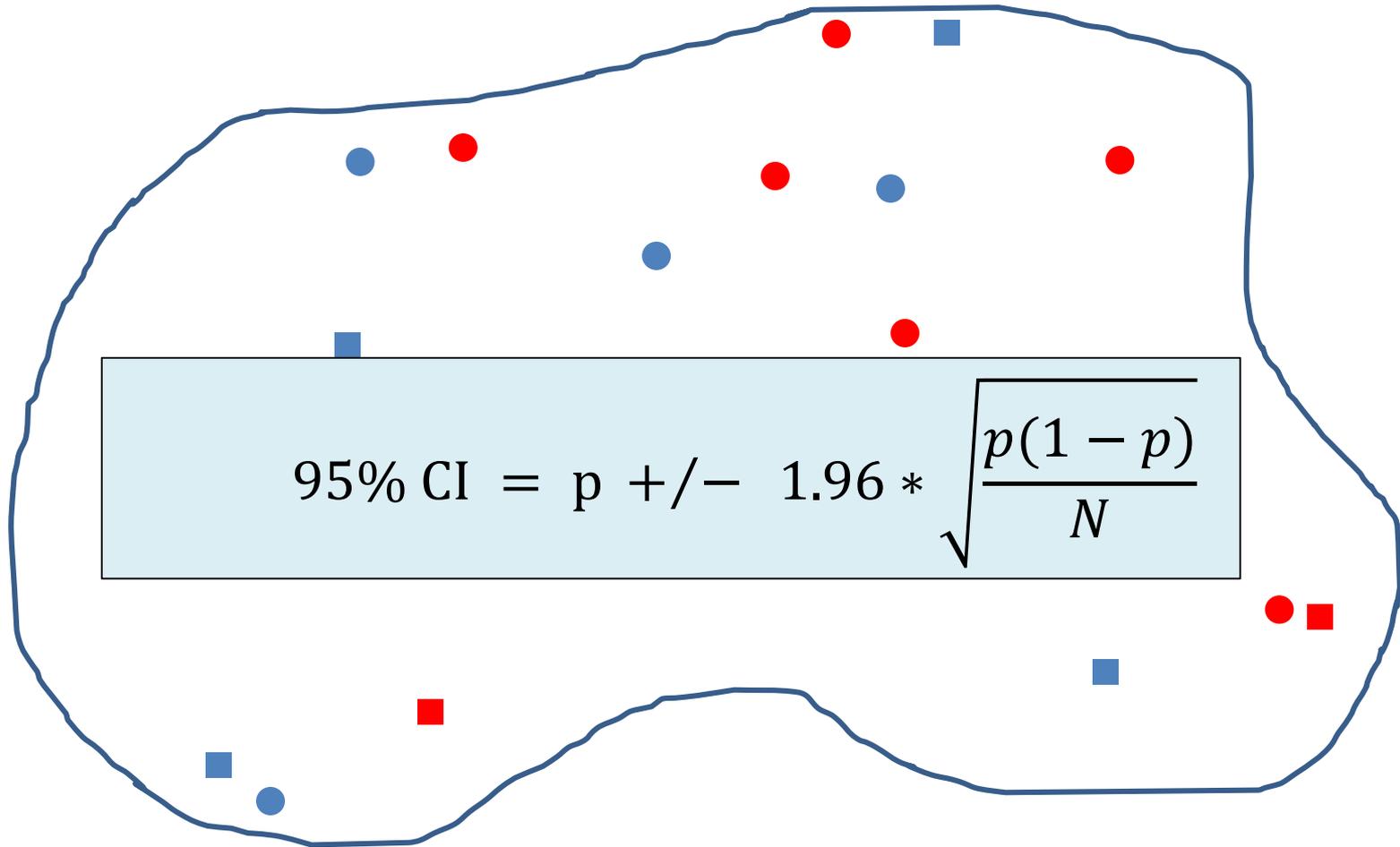
†Weighted for sampling fraction and difference in participation rates between census tracts.

# A note on confidence intervals

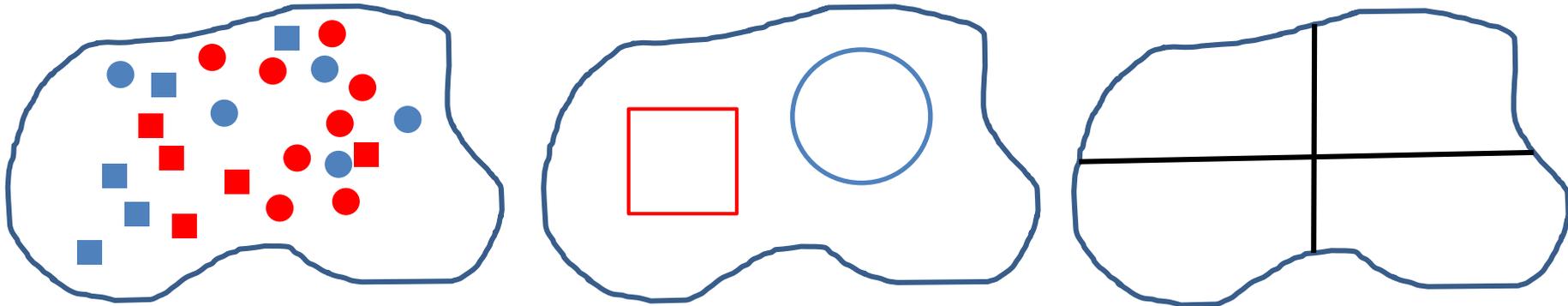
# Population



# Simple Random Sample



# Confidence Interval Methods for Samples



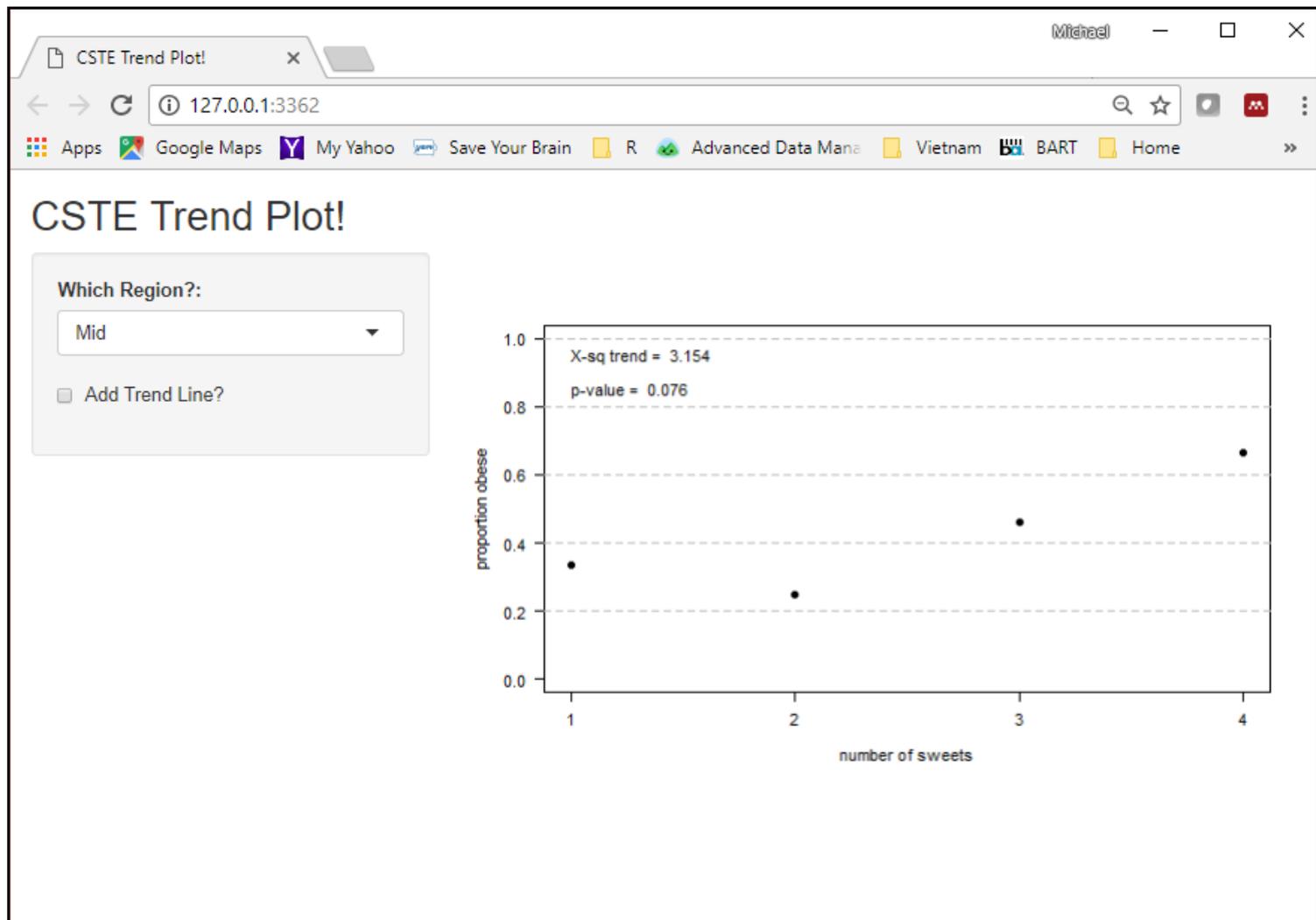
- Based on probability theory, the statistical issues and methods are well worked out, including for confidence intervals for simple random sampling, stratified sampling, cluster sampling and more
- Methods are well worked out for small numbers and other special situations

# What theory/methods for population-level data, as we often have in Public Health surveillance?

- 
- No clear theoretical basis upon which we calculate CIs for rates and proportions with population-level data like surveillance data
  - Metaphysical argument is that we are observing one “sample” of the unobservable complete world behind us
  - Other arguments suggest we know there is some “random” variation in most of things we observe, so therefore justified in using regular probability theory
  - (Of note, we rarely take into account the variation in the population denominator estimates (e.g. from census estimates/projections) but these do indeed have documented variability)

# What theory/methods for population-level data, as we often have in Public Health surveillance?

- In any case, some expression of the uncertainty and some statistical “tests” of differences in population-level rates and proportions **are essential** for communicating about these data and for making decisions
- Use of the “standard” tests appear to be the best alternative
- Might be reasonable to less precisely define confidence intervals in these situations (?)
  - **“Confidence intervals provide a guidepost for understanding if observed differences are (statistically) meaningful”**



<https://phdataviz.shinyapps.io/cstedemo/>

*Fusion Center for Strategic Development & External Relations*

# Resources

- Slides, spreadsheets, and R code for this presentation
  - [www.goo.gl/k9YmXJ](http://www.goo.gl/k9YmXJ)
- OpenEpi <http://www.openepi.com>
  - Web-based open source epidemiologic calculators and statistics for Public Health
- Epi Info Stat Calc <https://www.cdc.gov/epiinfo/index.html>
- R “home” page <https://www.r-project.org/>
- UNC FOCUS on Field Epidemiology -- Volume 3 - Data Analysis: Simple Statistical Tests
  - <https://sph.unc.edu/nciph/focus/>
- BMJ resource on statistical tests
  - <http://www.bmj.com/about-bmj/resources-readers/publications/statistics-square-one/8-chi-squared-tests>
- Good statistical information and tools
  - <http://stattrek.com/>
- Statistical Methods for Rates and Proportions, 2<sup>nd</sup> Edition. Joseph L. Fleiss. Wiley 1981.
  - The classic practical text on rates and proportion; newer 3<sup>rd</sup> Edition (with Bruce Levin, Myunghee Cho Paik) is likely excellent too

# QUESTIONS?



Let's Talk:

Michael C. Samuel, Dr.P.H

[Michael.Samuel@cdph.ca.gov](mailto:Michael.Samuel@cdph.ca.gov)

925-285-2926

“Calculation and comparison of rates and proportions are basic skills for epidemiologists. We’ve all done so, at least in grad school, but may be rusty around the edges. This basic webinar will review the fundamental concept of a rate and a proportion; review their calculation, including with confidence intervals; and review the assessment of trends in rates and proportions. We will demonstrate the calculation of these measures using free web-based “calculators” and with a little bit of code using R”

- After the webinar, participants will be able to:
  - State what rates and proportions are and why they are important
  - Calculate rate-related measures using free web applications
  - Calculate rate-related measures using R, that can be expanded for other “real world” production projects